3.5 Increasing, Decreasing, Max, and Min

So far we have been describing graphs using quantitative information. That’s just a fancy way to say that we’ve been using numbers. Specifically, we have described linear function graphs using the rate of change and initial value. Both are numerical data, however, at times it is more beneficial to describe functions in a qualitative manner. This means describing the qualities, non-numerical characteristics, of the function.

**Qualitative Data: Increasing or Decreasing**

Let’s start with the idea of an increasing or decreasing function. An *increasing function* roughly speaking is one that is going up when you look at it from left to right. This means that a *decreasing function* is one that is going down when you look at it from left to right. Let’s start by looking at linear functions.

**Linear functions**

*Increasing Linear Functions*

*Decreasing Linear Functions*

It is worth noting that in linear functions, whether it is increasing or decreasing is dependent on the slope of the line. Notice that those with positive slopes are increasing and those with negative slopes are decreasing.

However, how would we classify the graph to the left? It is possible to have a third option rather than just increasing or decreasing. It could be neither. The graph to the left is called **constant** because it stays the same from left to right.

It would be natural to then ask about a vertical line, but remember that a vertical line would not pass the vertical line test and therefore would not be a true function.
Non-linear functions

Linear functions are easy to identify as increasing, decreasing, or constant because they are a straight line. Non-linear functions might be both increasing and decreasing at different points on the graph. Consider the following graph of the function \( y = -x^2 + 8x - 10 \).

Looking at this graph from left to right, we see that the graph starts out increasing, but then it reaches a high point and start decreasing. So how do we classify this function?

Let’s see if we can isolate where the function is increasing and just look at that piece of the graph. Notice that the graph is increasing until it reaches the point at \( x = 4 \). That means we can say that the function is increasing when \( x < 4 \). In other words, whenever \( x \) is less than 4, the graph is increasing.

We can similarly see where the graph is decreasing which when \( x > 4 \). Notice that we don’t use greater than or equal to, just greater than because at the exact point \( x = 4 \) the graph is at the high point and neither increasing or decreasing. We’ll get to what that point is called in just a moment.

To the left is the graph of the function \( y = \frac{1}{3}x^3 - 4x \). Notice that it’s increasing, then decreasing, then increasing again. So we would say it is increasing when \( x < -2 \) and \( x > 2 \). It is decreasing in the interval \(-2 < x < 2\).
Qualitative Data: Maximum and Minimum

Another way that we can qualitatively describe a function graph is by identifying any maximum or minimum $y$ value that is achieved. For linear graphs this doesn’t make sense because the line will never have a maximum or minimum height. We’ll look at non-linear functions only for a max or min.

We say there is a **maximum** at $x$ (input) when the function has a higher $y$ value (output) than at any other input. Looking at our original non-linear example there is a maximum of $y = 6$. For this particular graph we can see that the maximum occurs at $x = 4$, but we generally list the output, or $y$ value, for a maximum.

Max: $y = 6$

We can do the same thing for a minimum. We say there is a **minimum** at $x$ (input) when the function has a lower $y$ value (output) than at any other input. In the example to left we see that it has a minimum of $y = -8$. It might also be worth noting that the minimum occurs at $x = -6$, but we list the minimum by its $y$ value.

Min: $y = -8$

Sometimes we have what are called **local** maximums or minimums. For example, in the graph to the left there is a local max of $y \approx 5$ at $x = -2$ and a local min of $y \approx -5$ at $x = 2$. They are called local because they are not the max or min for the whole graph, just the max or min in a small area.

Local max: $y \approx 5$  Local min: $y \approx -5$
Lesson 3.5

For each linear graph tell whether it is increasing, decreasing, or constant.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12.
For each non-linear graph tell where it is increasing and decreasing and identify any maximum, minimum, local maximum, or local minimum.