2.3A Comparing Linear Equations

In this section we’ll still be using the slope-intercept form of linear function equations. To be able to accurately compare functions, we may need to get the functions into the same representations, solve various representations of the functions, find the rates of change in each representation, and find the initial values in each representation.

Defining the Variables and Writing Equations

We want to make sure we can correctly identify the initial value in each situation so that we define our variables correctly. The key to equations is finding the lowest terms proportion ratio or slope.

Example 1: A company gained value at the rate of $200,000 per day after its IPO (initial public offering) of one million dollars.

What is actually being compared here? The hint is in the word “rate” which compares the value in dollars and the number of days after the company’s opening to the public stock market. That means we could define the variables and equation as follows:

\[ v = value \ of \ company \ in \ dollars \]
\[ d = days \ since \ the \ IPO \]
\[ v = 200,000d + 1,000,000 \]

Note that in this case the company was initially worth one million dollars. That’s why the one million is the initial value in this equation.

Example 2: To be a member at a gym you must pay a one-time $25 entry fee plus $10 per month.

Again, think of the slope or rate of change in this problem. How does the rate change? It changes by $10 per month. The entry fee does not change. That’s not part of the change. What changes is how much you pay when compared to the number of months you sign up for at the gym. That means we are comparing you amount you pay and the number of months. We define our variables and write the equation based on that as follows:

\[ c = cost \ of \ gym \ membership \]
\[ m = number \ of \ months \ for \ membership \]
\[ c = 10m + 25 \]

The $25 fee was the initial value because that is what you pay initially. That cost does not change over the number of months.
**Solving Equations**

Let's solve some linear function problems that have initial values using equations. Consider the gym membership situation from above. We could ask how much it would cost to be a member for one year. That would mean that \( m = 12 \) since one year is twelve months. We'll substitute and solve as follows:

\[
c = 10m + 25
\]

\[
c = 10(12) + 25
\]

\[
c = 120 + 25
\]

\[
c = 145
\]

This means that the cost for one year would be $145 total. We could also solve this problem if we knew the cost and not the number of months. For example, how many months of membership could you buy if you had $365? That would mean that \( c = 365 \), so we'll substitute and solve as follows:

\[
c = 10m + 25
\]

\[
365 = 10m + 25
\]

\[
365 - 25 = 10m + 25 - 25
\]

\[
365 - 25 = 10m + 25 - 25
\]

\[
340 = 10m
\]

\[
\frac{340}{10} = \frac{10m}{10}
\]

\[
\frac{340}{10} = \frac{40m}{40}
\]

\[
34 = m
\]

So we could buy 34 months of membership with $365.

**Finding the Initial Value (y-intercept) or Rate of Change (Slope)**

Pretend that a company gained value by $100 per day and after 7 days was worth $850. What was the initial value of the company? Let's start by defining the variables and writing an equation as best as we can. We know we are comparing the value of the company and the number of days. So we might do this:

\[
v = value \ of \ company \ in \ dollars
\]

\[
d = days \ since \ the \ IPO
\]

\[
v = 100d + b
\]

Notice that we don't know the initial value because that's what the question asked us to find. Therefore we'll just leave the variable \( b \) in the equation as a place holder for our initial value. However, we do know that after
7 days, or when \( d = 7 \), the company was worth $850, or \( v = 850 \). Let’s substitute those values in and solve for the initial value, or \( b \).

\[
v = 100d + b \\
850 = 100(7) + b \\
850 = 700 + b \\
850 - 700 = 700 + b - 700 \\
850 - 700 = 200 + b - 700 \\
150 = b
\]

Therefore the initial value of the company was $150.

Similarly we could solve for the slope, or rate of change, in a problem. Consider an appliance salesman who gets paid $50 every day plus some unknown amount for every appliance they sell that day. Let’s say the appliance salesman made $475 after selling 17 appliances. How much does he get paid per appliance? Again, we’ll define our variables and write the equation first.

\[
p = total \text{ pay} \\
a = number \text{ of appliances sold} \\
p = ma + 50
\]

Notice that the $50 every day is an initial value for the pay each day. That never changes. We also left the \( m \) in the equation because we don’t know the rate he gets per appliance. So let’s substitute what know, which is that when \( a = 17 \) then \( p = 475 \), and solve for \( m \).

\[
p = ma + 50 \\
475 = m \times 17 + 50 \\
475 = 17m + 50 \\
475 - 50 = 17m + 50 - 50 \\
475 - 50 = 17m + 50 - 50 \\
475 - 50 = 17m + 50 - 50 \\
475 - 50 = 17m + 50 - 50 \\
425 = 17m \\
\frac{425}{17} = \frac{17m}{17} \\
\frac{425}{17} = \frac{17m}{17} \\
\frac{425}{17} = \frac{17m}{17} \\
\frac{425}{17} = \frac{17m}{17} \\
m = 25
\]

This means that the salesman gets paid $25 per appliance that he sells.
Comparing Linear Functions using Equations

Now we have two values to compare in equations, the rate of change and the initial value. Let’s say that a restaurant wants to buy paper plates in bulk so they want to join a wholesale store (like Sam’s Club). There are three stores they could join:

**Store 1**: Charges a membership fee of $100 and charges $25 per bulk package of paper plates.

**Store 2**: Charges a membership fee of $50 and charges $30 per bulk package of paper plates.

**Store 3**: Charges a membership fee of $200 and charges $20 per bulk package of paper plates.

Let’s start by defining variables and writing equations for each store.

\[
c = \text{total cost} \\
p = \text{number of bulk packages of paper plates purchased}
\]

**Store 1**: \( c = 25p + 100 \)  
**Store 2**: \( c = 30p + 50 \)  
**Store 3**: \( c = 20p + 200 \)

Now it would be difficult to ask which store is cheapest because that depends on how many packages of paper plates you buy. However, we can ask which store has the cheapest rate for packages of paper plates. Which one does? Yes, Store 3 with the price of $20 per package.

Which store has the highest membership fee? That would be Store 3 as well since it charges $200.

Which store would give the overall cheapest price if the restaurant needed to buy two packages of paper plates? What if it were five packages? What if it were ten packages? Substitute each of those values in for \( p \) and solve for \( c \).

**Two packages**

**Store 1**: \( c = 25(2) + 100 \)  
\[ c = 150 \]

**Store 2**: \( c = 30(2) + 50 \)  
\[ c = 110 \]

**Store 3**: \( c = 20(2) + 200 \)  
\[ c = 240 \]

In this case Store 2 is the best store to buy from.
Five packages

Store 1: $c = 25(5) + 100$  
Store 2: $c = 30(5) + 50$  
Store 3: $c = 20(5) + 200$

$c = 225$  
$c = 200$  
$c = 300$

In this case Store 2 is the cheapest.

Twenty packages

Store 1: $c = 25(20) + 100$  
Store 2: $c = 30(20) + 50$  
Store 3: $c = 20(20) + 200$

$c = 600$  
$c = 650$  
$c = 600$

In this case there is a tie for the cheapest store between Store 1 and Store 3. You should be able to extrapolate (think ahead) and see that if the restaurant needs to buy more than twenty packages of paper plates in bulk, Store 3 will always be cheaper. Can you explain why?
Lesson 2.3A

Use the given equation to solve the questions.

1. If a roller coaster starts 12 meters above the ground and climbs 2 meters every second \( s \), the roller coaster's height \( h \) would be based on the equation \( h = 2s + 12 \). How long would it take to reach the top of the hill that is 80 meters above the ground?

2. If it is going to cost you $525 dollars to start a lawn care business with your friend, but you will earn an average of $73 for every 4 yards \( y \), your profit \( p \) is based on the equation \( p = \frac{73}{4}y - 525 \). How much profit would you make if you were scheduled to mow 48 yards the first summer?

3. The CMS dance team is hosting a car wash fundraiser and charging $3 per car. If the dance team washed a total of 14 cars \( c \), how much money \( m \) did the team make if you followed the equation \( m = 3c \)?

4. If you spent $10.35 total \( t \) purchasing songs online for $1.15 each, how many songs did you buy \( s \) if you followed the equation \( t = 1.15s \)?

5. Your running pace is 1 mile every 8 minutes. If you ran a distance \( d \) of 5.5 miles, how many minutes \( m \) were you running if you followed the equation \( d = \frac{1}{8}m \)?

6. If CMS orders 25 cartons of milk \( m \) plus 1 for every 3 students \( s \) eating lunch, the number of cartons of milk they order is based on the equation \( m = \frac{1}{3}s + 25 \). How many cartons of milk should they order if there are 399 students eating lunch today?
Define variables and create an equation to solve the following questions.

7. At the Charleston Bowling Lanes, it costs $2 to rent shoes plus $1.50 per game of bowling. How many games would you be able to bowl for $11?

8. You have already read 173 pages in the first book of the Twilight series. If you read about 65 pages every 2 nights, how long will it take you to finish the book that is 498 pages long?

9. To make the perfect pizza, there should be 4 pieces of pepperoni for every 3 slices of mushrooms. If you put 24 pieces of pepperoni, how many mushroom slices should you use?

10. The recipe for iced coffee at Starbucks suggests 2 parts milk for every 3 parts coffee. If a venti (the largest size) requires 12 ounces of coffee, how many ounces of milk should be added?

11. Your parents put a down payment on your car, but they are requiring you to pay the monthly payment of $85. If you will have to pay a total of $2125 for the car, how long will it take you to pay it off?

12. The local humane society receives $45 for every dog they give up for adoption. If they spent $920 on supplies, how much profit will they make if they have 24 dogs to give up for adoption?

13. John has the sequence 0, 5, 10, 15, 20, 25 ... What is the 15th term in the sequence?

14. Joe has the sequence 9, 7, 5, 3, 1, -1 ... What is the 10th term in the sequence?

15. Jill has the sequence 3, 3.5, 4, 4.5, 5, 5.5, 6 ... What is the 20th term in the sequence?

16. Jane has the sequence -5, -4.25, -3.5, -2.75, -2, -1.25 ... What is the 10th term in the sequence?
Answer the following questions comparing linear function equations and descriptions.

Dr. Kal is studying how age and gender affect calorie expenditure. Here is the information about the number of calories burned (c) based on the number of miles (m) walked in a day.

<table>
<thead>
<tr>
<th></th>
<th>Burns 1390 calories plus 1040 calories from walking 10 miles</th>
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</thead>
<tbody>
<tr>
<td>Paul (25)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Burns 1305 calories plus 220 calories from walking 2 miles</th>
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<tbody>
<tr>
<td>Ishmael (58)</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Calorie expenditure is based on the equation c = 98m + 1225</th>
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<tbody>
<tr>
<td>Jerika (31)</td>
<td></td>
</tr>
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<table>
<thead>
<tr>
<th></th>
<th>Calorie expenditure is based on the equation c = (\frac{205}{2}m + 1189)</th>
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</thead>
<tbody>
<tr>
<td>Pamela (62)</td>
<td></td>
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</table>

17. Who burns the most calories per mile, and how do you know?

18. Who burns the least calories per mile, and how do you know?

19. Who burns the most calories without walking, and how do you know?

20. How far would each person have to walk (to the nearest hundredth) to burn 2000 calories?

21. If each person walks 2 miles, who burns the most calories for that day?

22. If each person walks 10 miles, who burns the most calories for that day?

Answer the following questions comparing linear function equations and descriptions.

You are deciding which gas company to choose as you travel across the country on a long vacation with your family. Here is the information about the cost (c) for gallons of gas (g) for each company.

<table>
<thead>
<tr>
<th></th>
<th>Charges $4.01 for each gallon of gas</th>
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<tbody>
<tr>
<td>Gas Up</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Charges $81 for 20 gallons of gas</th>
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<tbody>
<tr>
<td>Automart</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Cost is modeled by the equation c = 4.03g</th>
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<tbody>
<tr>
<td>The Fuel Shop</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Cost is modeled by the equation c = 4.10g</th>
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</thead>
<tbody>
<tr>
<td>Full Tank</td>
<td></td>
</tr>
</tbody>
</table>

23. Which company charges the most per gallon of gas? How do you know?

24. Which company charges the least per gallon of gas? How do you know?

25. How much would each company charge you for 12 gallons of gas? Which is the cheapest?